

The z-Transform

Electrical Engineering

Laplace for discrete time, with the unit circle as its stability boundary.

The z-transform does for discrete-time systems what the Laplace transform does for continuous ones.

$$X(z) = \sum x[n] z^{-n} \cdot z = \exp(sT)$$

One-sample delay

The basic block.

z^{-1}

Pole inside the circle

$|z| < 1$.

Stable

$a^n u[n]$

For $|z| > |a|$.

$z/(z - a)$

THE COMMON MISTAKE

✗ Testing discrete stability in the left half plane

✓ Testing it against the unit circle

The map $z = \exp(sT)$ sends the left half of the s plane to the inside of the unit circle, so the boundary changes shape entirely.

Sampling wraps the frequency axis onto a circle, so a discrete system's response repeats every sampling frequency.