

Uniform Convergence

Mathematics

What uniform convergence preserves and pointwise convergence loses.

A sequence of functions f_n tending to f .

Question	Pointwise	Uniform
Definition	N depends on x	One N serves every x
Limit of continuous f_n	May be discontinuous	Is continuous
Swap limit and integral	Not in general	Valid on $[a, b]$
Swap limit and derivative	No	Needs f_n' uniform too
Boundedness carried over	No	Yes

x^n on $[0, 1]$ converges pointwise to a discontinuous limit. On $[0, c]$ with $c < 1$ the same convergence is uniform.