

Trig Substitution

Mathematics

Which substitution clears which square root, and what it leaves behind.

Take $a > 0$ and θ in the principal range.

Expression	Substitute	Becomes
$\sqrt{a^2 - x^2}$	$x = a \sin \theta$	$a \cos \theta$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta$	$a \sec \theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta$	$a \tan \theta$
$a^2 + x^2$, no root	$x = a \tan \theta$	$a^2 \sec^2 \theta$
$x^2 + bx + c$	Complete the square	Then use a row above

Convert back with a right triangle, not by inverting the substitution blindly. $\sin \theta = x/a$ fixes every other ratio at once.