Taylor Series

Mathematics

Matching a function with the polynomial whose derivatives agree at a point.

Near a, a smooth function is approximated by the polynomial whose derivatives at a match its own.

$$f(x) = \Sigma f^{(n)}(a)(x - a)^n / n!$$

e^x about 0 — $1 + x + x^2/2 + x^3/6 + \dots$

Valid for every x.

sin x about 0 — $x - x^3/6 + x^5/120 - \dots$

Odd powers only.

1/(1 − x) about 0 — $1 + x + x^2 + x^3 + \dots$

Only for $|x| < 1$.

THE COMMON MISTAKE

× **Assuming a smooth function equals its series**

✓ **Checking that the remainder tends to zero**

The function $e^{(-1/x^2)}$, extended by $f(0) = 0$, is smooth with every derivative zero at 0, so its Taylor series is 0 and matches nowhere else.

Maclaurin is the case a = 0. The radius of convergence is fixed by the nearest singularity, in the complex plane if not the real line.