

The Symmetric Group

Mathematics

Permutations in cycle notation, with order and parity read straight off.

S_n is the group of all rearrangements of n symbols.

Structure

S_n contains every permutation of n symbols

$$|S_n| = n!$$

Non-abelian for every $n \geq 3$

Every element is a product of disjoint cycles

Disjoint cycles commute with each other

Reading a permutation

The order of a single cycle is its length

Order overall is the lcm of cycle lengths

Parity, even or odd, is well defined

A_n is the even permutations, of index 2

Cayley: every group embeds in some S_n

A_n is simple for every $n \geq 5$. That single fact is why the general quintic has no solution by radicals.