Supremum & Infimum

Mathematics

Least upper bounds, and the property that separates the reals from the rationals.

The supremum of a set is its least upper bound; the infimum is its greatest lower bound.

SUP OF (0, 1)	MAX OF (0, 1)	INF OF {1/N}
1, not in the set	Does not exist	0, never attained

- sup S is an upper bound, and no smaller one exists
- sup S need not lie in S; a maximum must
- sup of the open interval (0, 1) is 1, not attained
- Every nonempty set bounded above has a sup in □
- □ fails that: the sup of $\{x : x^2 < 2\}$ is not rational
- By convention sup ∅ = $-\infty$ and inf ∅ = $+\infty$

LOOK FOR IT

The sup of $\{x$ □ □ $: x^2 < 2\}$ is $\sqrt{2}$, which is exactly why □ is incomplete.

The least upper bound property is what distinguishes □ from □. Every theorem about the real line rests on it somewhere.