

The Subspace Test

Mathematics

Three checks that replace verifying all ten vector space axioms.

A nonempty subset W of V is a subspace exactly when it is closed under addition and under scalar multiplication.

$$0 \in W \cdot u, v \in W \Rightarrow u + v \in W \cdot \lambda u \in W$$

Lines through 0 in $\mathbb{R}^2$

Closed both ways.

A subspace

$y = x + 1$ in $\mathbb{R}^2$

Misses the origin.

Not a subspace

Solutions of $Ax = 0$

This is the null space.

A subspace

THE COMMON MISTAKE

✗ **Checking only that W is nonempty**

✓ **Checking $0 \in W$ and both closures**

Every subspace contains the zero vector, because taking $\lambda = 0$ forces it. A set missing the origin fails before any other test is reached.

Solutions of $Ax = 0$ always form a subspace. Solutions of $Ax = b$ never do, unless b happens to be zero.