

The Subgroup Test

Mathematics

One condition that replaces re-checking every group axiom.

A nonempty subset H of G is a subgroup exactly when ab^{-1} lies in H for every a and b in H .

$$H \neq \emptyset \text{ and } a, b \in H \Rightarrow ab^{-1} \in H$$

Even integers inside

Closed under $a - b$.

A subgroup

Odd integers inside

It misses 0.

Not a subgroup

Finite closed subset

Finite gives inverses.

A subgroup

THE COMMON MISTAKE

✗ **Checking closure under the operation only**

✓ **Also checking inverses, or using ab^{-1}**

For an infinite subset closure alone is not enough: the positive integers are closed under addition and contain no inverses at all.

For a finite nonempty subset, closure under the operation is sufficient on its own. Infinite subsets need the inverse condition too.