Solving ODEs by Laplace

The full route from an initial-value problem to $y(t)$.

Mathematics

For a linear ODE with constant coefficients and given initial values.

1. **Transform every term**
   The derivative rules pull the initial conditions in automatically.

2. **Substitute the initial values**
   Put $y(0)$ and $y'(0)$ in now. No arbitrary constants will appear later.

3. **Solve for $Y(s)$**
   The transformed equation is algebraic in $Y(s)$. Just rearrange it.

4. **Split into standard forms**
   Partial fractions, then complete the square where roots are complex.

5. **Invert term by term**
   Read each piece backwards from the table to recover $y(t)$.

Its real advantage is discontinuous forcing: step and impulse inputs transform cleanly where the classical methods need cases.