

Singular Value Decomposition

Mathematics

The factorisation that exists for every matrix, square or not.

Every real $m \times n$ matrix factors into an orthogonal matrix, a diagonal one and another orthogonal one.

SHAPE OF U

$m \times m$, orthogonal

SHAPE OF Σ

$m \times n$, diagonal

SHAPE OF V

$n \times n$, orthogonal

- Written $A = U \Sigma V^T$, with U and V orthogonal
- Σ holds the singular values $\sigma_1 \geq \sigma_2 \geq \dots \geq 0$
- They are the square roots of eigenvalues of $A^T A$
- Rank is the count of nonzero singular values
- Truncating Σ gives the best low-rank approximation
- It exists for every matrix, singular or not

LOOK FOR IT

A 2×3 matrix of rank 2 has exactly two nonzero singular values.

The condition number is the largest singular value over the smallest. It bounds how much solving $Ax = b$ can amplify error in b .