

Convergence Tests

Mathematics

Which test to reach for, in the order worth trying them.

Apply in order. Stop at the first that settles it.

Test	Use when	Verdict
nth term	Always, first	a_n not $\rightarrow 0$ means diverges
Geometric	Ratio r is constant	Converges iff $ r < 1$
p-series	Terms are $1/n^p$	Converges iff $p > 1$
Ratio	Factorials or powers	$L < 1$ converges, $L > 1$ not
Comparison	Positive terms only	Bounded by a known series
Alternating	Signs alternate	Needs decreasing and $\rightarrow 0$
Integral	f positive, decreasing	Series and integral agree

The ratio test is inconclusive at $L = 1$, which is exactly where the p-series live. $\sum 1/n$ diverges while $\sum 1/n^2$ converges.