Sequence Convergence

Mathematics

What convergence requires, and the intuitions that do not survive it.

A sequence converges to L when past some index N every later term lies within ε of L, for whatever ε you are given.

$$\forall \varepsilon > 0 \ \exists N : n > N \Rightarrow |a_n - L| < \varepsilon$$

$a_n = 1/n$ — **Converges to 0**

Take N > 1/ε.

$a_n = (-1)^n$ — **Diverges**

Two limit points.

$a_n = n$ — **Diverges to infinity**

Unbounded.

THE COMMON MISTAKE

× Treating 'gets closer' as convergence

✓ Requiring every ε to be beaten eventually

$a_n = 1 + 1/n$ moves closer to 0 at every step and converges to 1. Approaching a number is not the same as converging to it.

A convergent sequence is bounded and its limit is unique. Bounded does not imply convergent — $(-1)^n$ is the standard counterexample.