

Separation of Variables

Mathematics

Reducing a PDE to two ODEs, then rebuilding the solution from its modes.

For a linear homogeneous PDE on a bounded domain.

1

Assume a product solution

Try $u(x, t) = X(x)T(t)$ and substitute it into the equation.

2

Separate the variables

Divide until each side depends on one variable, then set both to $-\lambda$.

3

Solve the eigenvalue problem

The boundary conditions select the allowed λ and the modes X_n .

4

Solve for the time factor

Each λ_n gives one T_n . For the heat equation these decay.

5

Superpose and fit the data

Sum the modes, then use Fourier coefficients to match the initial state.

It needs linearity, homogeneous boundary conditions and a separable domain. Inhomogeneous terms must be dealt with first, separately.