# Second-Order Linear ODEs

Mathematics

The three root cases of the auxiliary equation, and their solutions.

*For $ay'' + by' + cy = 0$, try $y = e^{(mx)}$.*

| Discriminant | Roots m | General solution |
|---|---|---|
| $b^2 - 4ac > 0$ | Real and distinct | $A\,e^{(m_1x)} + B\,e^{(m_2x)}$ |
| $b^2 - 4ac = 0$ | Real and repeated | $(A + Bx)\,e^{(mx)}$ |
| $b^2 - 4ac < 0$ | $p \pm qi$ | $e^{(px)}(A \cos qx + B \sin qx)$ |
| $b = 0$ with $ac > 0$ | Purely imaginary $\pm qi$ | $A \cos qx + B \sin qx$ |

**The three cases are overdamped, critically damped and underdamped. Critical damping returns to rest fastest without overshooting.**