

Rings, Domains & Fields

Mathematics

The ladder of ring structures, one added requirement per rung.

Each row adds one requirement to the row above it.

Structure	Adds	Example
Ring	Abelian +, associative $\times$	2×2 matrices over $\mathbb{Z}$
Commutative ring	$ab = ba$ throughout	$\mathbb{Z}$, and $\mathbb{Z}[x]$
Integral domain	No zero divisors	$\mathbb{Z}$
Field	Every nonzero element a unit	$\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$
Division ring	Units without commutativity	The quaternions

Every field is an integral domain; the converse fails, with $\mathbb{Z}$ the standard case. Every finite integral domain is in fact a field.