Riemann Integrability

Mathematics

When upper and lower sums meet, and when they never do.

f is Riemann integrable on [a, b] when some partition makes the upper and lower sums arbitrarily close.

[illegible] $\varepsilon > 0$ [illegible] partition $P : U(f, P) - L(f, P) < \varepsilon$

f continuous on [a, b] — **Integrable**

Uniform continuity.

f monotone on [a, b] — **Integrable**

Even with jumps.

Indicator of [illegible] on [0, 1] — **Not integrable**

$U = 1$ and $L = 0$.

THE COMMON MISTAKE

× **Assuming bounded implies integrable**

✓ **Bounded plus continuity almost everywhere**

Lebesgue's criterion: a bounded f is Riemann integrable exactly when its set of discontinuities has measure zero.

Riemann handles bounded functions on bounded intervals. Lebesgue extends the theory to limits, which is why analysis moved on.