

Recurrence Relations

Mathematics

Solving a linear recurrence with constant coefficients, in closed form.

For a_n defined from a_{n-1} and a_{n-2} with fixed coefficients.

1

Write the characteristic equation

For $a_n = c_1 a_{n-1} + c_2 a_{n-2}$, the equation is $r^2 = c_1 r + c_2$.

2

Solve for the roots

Distinct roots r_1 and r_2 give the general form $A r_1^n + B r_2^n$.

3

Handle a repeated root

A double root r gives $(A + Bn) r^n$ rather than two exponentials.

4

Add a particular solution

For a non-homogeneous term, guess a form matching it and solve.

5

Fit the initial conditions

Substitute a_0 and a_1 last. Only now are A and B determined.

Fibonacci gives $r^2 = r + 1$, so a_n grows like ϕ^n , where ϕ is the golden ratio. The other root is negative and shrinks away.