

Rank-Nullity Theorem

Mathematics

The conservation law relating the image and the kernel of a linear map.

For a linear map T from V to W with V finite-dimensional, the rank plus the nullity equals $\dim V$.

$$\dim V = \text{rank}(T) + \text{nullity}(T)$$

A is 3×5 with rank 3
5 – 3.

Nullity is 2

T is injective
Trivial kernel.

Nullity is 0

3×3 matrix of rank 2
So not invertible.

$\det(A) = 0$

THE COMMON MISTAKE

✗ Reading $\dim V$ as the codomain's dimension

✓ $\dim V$ is the dimension of the domain

Rank lives in W and nullity lives in V , but the sum is $\dim V$. Using $\dim W$ gives the wrong count whenever the map is not square.

Row rank always equals column rank. Rank is the number of pivots, and that number is the same read either way.