

Quotient Groups

Mathematics

Constructing G/H , and why normality is what makes it well defined.

Building G/H when H is a normal subgroup of G .

1

Check H is normal

$gHg^{-1} = H$ for every g in G . Without this the construction fails.

2

Take cosets as elements

The elements of G/H are the cosets gH , not the elements of G .

3

Define the operation

$(aH)(bH) = abH$. Normality is exactly what makes this well defined.

4

Identify identity and inverse

H itself is the identity coset, and the inverse of aH is $a^{-1}H$.

5

Read off the order

For a finite group $|G/H| = |G|/|H|$, straight from Lagrange.

$\mathbb{Z}/n\mathbb{Z}$ is the integers modulo n . Quotienting collapses H to a single point and keeps only the structure that survives that.