

Quantifiers & Negation

Mathematics

How to negate a quantified statement without changing its meaning.

Push the negation inward; each quantifier flips.

Statement	Its negation
$\forall x P(x)$	$\exists x \neg P(x)$
$\exists x P(x)$	$\forall x \neg P(x)$
$\forall x \forall y P(x, y)$	$\exists x \exists y \neg P(x, y)$
$\forall x (P(x) \rightarrow Q(x))$	$\exists x (P(x) \wedge \neg Q(x))$
Every swan is white	Some swan is not white

Order matters. 'For every x there is a y' is far weaker than 'there is a y for every x' — in one the y may depend on x, in the other it may not.