

Radius of Convergence

Mathematics

Finding where a power series converges, endpoints included.

For a series in powers of $(x - a)$ with coefficients c_n .

1 Apply the ratio test

Take the limit L of $|c_{n+1} / c_n|$ as n grows, when that limit exists.

2 Invert to get R

$R = 1/L$. If $L = 0$ then R is infinite; if L is infinite then $R = 0$.

3 Write the open interval

The series converges absolutely whenever $|x - a| < R$.

4 Test both endpoints

At $|x - a| = R$ the ratio test is silent. Substitute and test directly.

5 State the full interval

The endpoints may be both included, both excluded, or one of each.

Convergence is uniform on any closed interval strictly inside the radius, so term-by-term differentiation is valid there.