Polynomial Rings

Mathematics

What F[x] inherits from F, and how irreducibility builds new fields.

R[x] is the ring of polynomials in x whose coefficients are drawn from a ring R.

□[X] MODULO (X^2 + 1)	ROOTS OF A DEGREE-N F	IDEALS OF F[X]
Isomorphic to □	**At most n, over a field**	**Every one is principal**

- R[x] is an integral domain whenever R is
- Over a domain, deg(fg) = deg f + deg g
- Over a field, division with remainder always works
- F[x] is a principal ideal domain and a UFD
- f(a) = 0 exactly when (x − a) divides f
- A degree-n polynomial has at most n roots in a field
- If f is irreducible then F[x] modulo (f) is a field

LOOK FOR IT

x^2 + 1 is irreducible over □ and splits into two factors over □.

Irreducibility depends on the coefficient ring. x^2 − 2 is irreducible over □ and factors immediately over □.