Pigeonhole Principle

Mathematics

The cheapest existence argument in maths, plus its generalised form.

If n objects are placed into m boxes and n is greater than m, then some box holds at least two objects.

n items, m boxes ⇒ some box holds ≥ ⌈n/m⌉

13 people — **Two share a birth month**

13 > 12.

5 points in a unit square — **Two within √2/2**

Quarter the square.

n + 1 taken from 1 to 2n — **Two are coprime**

A consecutive pair.

THE COMMON MISTAKE

× Concluding which box is the full one

✓ Concluding only that some box is

The principle is a pure existence statement. It names no box and builds nothing, which is exactly why it costs so little to apply.

The generalised form: with n objects in m boxes, some box must hold at least ⌈n/m⌉ of them.