

Phase Plane Classification

Mathematics

Reading the behaviour near an equilibrium off the eigenvalues.

For $x' = Ax$ in the plane, near the origin.

Eigenvalues of A	Type	Stability
Real, both negative	Node	Stable
Real, both positive	Node	Unstable
Real, opposite signs	Saddle	Unstable
Complex, negative real part	Spiral	Stable
Complex, positive real part	Spiral	Unstable
Purely imaginary	Centre	Neutrally stable

Stable exactly when both eigenvalues have negative real part, which for a 2×2 system means $\text{trace} < 0$ and $\text{determinant} > 0$.