

Permutations vs Combinations

Mathematics

The single question that separates the two, with the formulas either side.

Permutations

- Order matters
- $P(n, r) = n! / (n - r)!$
- Arrangements, rankings, passwords
- ABC and CBA are different
- $n!$ for the whole set
- With repetition allowed: n^r

Combinations

- Order does not matter
- $C(n, r) = n! / (r!(n - r)!)$
- Committees, hands, subsets
- ABC and CBA are the same
- 2^n subsets in total
- Symmetry: $C(n, r) = C(n, n - r)$

THE DIFFERENCE THAT MATTERS

$C(n, r) = P(n, r) / r!$ — you divide out the $r!$ orderings that were counted separately.

If swapping two chosen items gives a different outcome, it is a permutation. If it gives the same outcome, it is a combination.