Classifying PDEs

Mathematics

Elliptic, parabolic or hyperbolic, and what each type implies.

A second-order linear PDE in $A u_{xx} + B u_{xy} + C u_{yy}$ is classified by the sign of $B^2 - 4AC$.

LAPLACE, $\nabla^2 U = 0$	HEAT, $U_T = K U_{XX}$	WAVE, $U_{TT} = C^2 U_{XX}$
Elliptic	**Parabolic**	**Hyperbolic**

- Negative: elliptic, like Laplace's equation
- Zero: parabolic, like the heat equation
- Positive: hyperbolic, like the wave equation
- Elliptic problems are steady states, with no time direction
- Parabolic equations smooth their data out as time runs
- Hyperbolic equations propagate at a finite speed
- The type can change from one region to another

LOOK FOR IT

The equation $u_{xx} + y\, u_{yy} = 0$ changes type as it crosses the line $y = 0$.

The type dictates which data make the problem well posed: elliptic needs values on a closed boundary, hyperbolic needs initial data.