

Partial Fractions

Mathematics

Splitting a rational function into pieces you already know how to integrate.

For a rational function $P(x)/Q(x)$.

1 Check the degrees

If $\deg P$ is at least $\deg Q$, divide first. The fraction must be proper.

2 Factor the denominator

Into linear and irreducible quadratic factors over the reals.

3 Write the template

$A/(x - a)$ per linear factor; $(Bx + C)/(x^2 + px + q)$ per quadratic.

4 Handle repeated factors

A factor $(x - a)^n$ needs one term for every power from 1 up to n .

5 Solve for the constants

Clear denominators, then substitute the roots or compare coefficients.

Worked: $1/(x^2 - 1) = \frac{1}{2}[1/(x - 1) - 1/(x + 1)]$, so the integral is $\frac{1}{2} \ln|(x - 1)/(x + 1)| + C$.