Partial Derivatives

Mathematics

Gradients, directional derivatives and what they tell you geometrically.

Calculus in more than one variable.

The objects

- ∂f/∂x — vary x, hold every other fixed
- ∇f = (∂f/∂x, ∂f/∂y), the gradient
- Directional: ∇f · u, with u a unit vector
- Hessian — the matrix of second partials
- Jacobian — the matrix of first partials

What they tell you

- ∇f points along the steepest ascent
- |∇f| is that steepest rate of change
- ∇f = 0 at every critical point
- ∇f is normal to the level curve f = c
- Mixed partials agree if both are continuous

Clairaut's theorem: the two mixed second partials are equal whenever both are continuous near the point.