Open & Closed Sets

Mathematics

The two definitions, their closure properties, and why they are not opposites.

Open and closed are not opposites of each other.

Open sets

- Every point has a ball lying inside
- Arbitrary unions stay open
- Finite intersections stay open
- (0, 1) and ℝ itself are open
- Complement of a closed set

Closed sets

- Contains all of its limit points
- Arbitrary intersections stay closed
- Finite unions stay closed
- [0, 1] and any finite set are closed
- Complement of an open set

ø and ℝ are both open and closed at once. [0, 1) is neither. 'Not open' never licenses the conclusion 'closed'.