The Mean Value Theorem

Mathematics

The theorem behind almost every result about derivatives, hypotheses included.

If f is continuous on [a, b] and differentiable on (a, b), some c in (a, b) has f′(c) equal to the average rate.

$$f'(c) = (f(b) - f(a)) / (b - a)$$

$f(x) = x^2$ on [0, 2] — **c = 1**

$f'(1) = 2$, the slope.

Rolle: f(a) = f(b) — **$f'(c) = 0$**

The special case.

$f(x) = |x|$ on [−1, 1] — **No such c exists**

Not differentiable at 0.

THE COMMON MISTAKE

× **Applying it to $f(x) = 1/x$ on [−1, 1]**

✓ **Checking continuity on the closed interval first**

1/x is not continuous at 0, so the theorem says nothing at all. The hypotheses are not decoration — they are what buys the conclusion.

Continuity is required on the closed interval, differentiability only on the open one. The endpoints need no derivative.