

Maximum Likelihood

Mathematics

The estimation method behind most of statistics, in five steps.

Choose the parameter making the observed data most probable.

1

Write the likelihood

$L(\theta)$ is the joint density of the data, read as a function of θ .

2

Take logarithms

Products become sums, and the maximising θ is unchanged.

3

Differentiate

Set the derivative of the log-likelihood to zero for the score equation.

4

Solve for θ

Analytically where possible, numerically where it is not.

5

Confirm it is a maximum

The second derivative must be negative, and check the boundary too.

MLE is consistent and asymptotically efficient, but can be biased in small samples: the MLE of a variance divides by n , not $n - 1$.