

Matrix Multiplication

Mathematics

How the product is formed, which shapes are legal, and why order matters.

The (i, j) entry of AB is row i of A dotted with column j of B . The inner dimensions must match.

$$(AB)_{ij} = \sum_k a_{ik} b_{kj}$$

$(2 \times 3)(3 \times 4)$

Inner 3s agree.

A 2×4 matrix

$(2 \times 3)(2 \times 3)$

$3 \neq 2$.

Undefined

AB against BA

Not commutative.

Usually different

THE COMMON MISTAKE

✗ Multiplying entrywise, a_{ij} times b_{ij}

✓ Row of A against column of B

The entrywise product is the Hadamard product, a different operation. The matrix product composes linear maps, which is why it is not commutative.

The product is associative and distributive but not commutative, and $AB = 0$ does not force $A = 0$ or $B = 0$.