The Matrix Inverse

Mathematics

When an inverse exists, how to find one, and the order rule for products.

A square matrix is invertible exactly when its determinant is nonzero, and then the inverse is unique.

$$A^{-1} = \text{adj}(A) / \det(A)$$

2×2 [a b; c d] — **[d −b; −c a]/(ad − bc)**

Swap, negate, divide.

det(A) = 0 — **No inverse exists**

A singular matrix.

$(AB)^{-1}$ — **$B^{-1}A^{-1}$**

The order reverses.

THE COMMON MISTAKE

× $(AB)^{-1} = A^{-1}B^{-1}$

✓ $(AB)^{-1} = B^{-1}A^{-1}$

Inverting reverses a composition, the way you remove shoes before socks. The two agree only when the matrices commute.

Beyond 3×3, solve Ax = b by row reduction rather than inverting. Computing the inverse is slower and numerically worse.