

Logical Equivalences

Mathematics

The rewrites that are valid, beside the ones that look valid and are not.

$\equiv$ means the two have identical truth tables.

Valid rewrites

$$p \rightarrow q \equiv \neg p \vee q$$

$$\text{Contrapositive: } p \rightarrow q \equiv \neg q \rightarrow \neg p$$

$$p \Leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$\text{Double negation: } \neg \neg p \equiv p$$

$$\text{De Morgan: } \neg(p \wedge q) \equiv \neg p \vee \neg q$$

Not equivalent to $p \rightarrow q$

$$\text{Converse: } q \rightarrow p$$

$$\text{Inverse: } \neg p \rightarrow \neg q$$

$$\neg(p \rightarrow q) \equiv p \wedge \neg q, \text{ not } p \rightarrow \neg q$$

$$q \rightarrow p \text{ follows only if } p \Leftrightarrow q$$

$$\neg p \vee \neg q \text{ is not } \neg(p \wedge q)$$

A statement and its contrapositive are equivalent. A statement and its converse are not, and confusing the two is the classic error.