Systems of ODEs

Mathematics

Solving a constant-coefficient system through the eigenvalues of its matrix.

For $x' = Ax$ with A constant, each eigenvalue and eigenvector pair supplies one exponential solution.

$$x = \Sigma\, c_i\, v_i\, e^{(\lambda_i t)}, \text{ where } Av_i = \lambda_i v_i$$

Distinct real eigenvalues — **n independent solutions**

Superpose them.

$\lambda = p \pm qi$ — **$e^{(pt)}$ with cos and sin**

Take real parts.

Repeated λ, one vector — **Need $t\, e^{(\lambda t)}$ terms**

Generalised vectors.

THE COMMON MISTAKE

× **Assuming n eigenvalues give n eigenvectors**

✓ **Checking the geometric multiplicity**

A defective matrix has fewer independent eigenvectors than eigenvalues, and the missing solutions carry extra factors of t.

Any higher-order linear ODE becomes a first-order system by setting $x_1 = y$, $x_2 = y'$, and so on down the derivatives.