

# Limits & Continuity

Mathematics

The limit laws, and the three conditions continuity actually requires.

*A limit describes approach, not arrival.*

## Limit laws (both limits must exist)

Sum:  $\lim(f + g) = \lim f + \lim g$

Product:  $\lim(fg) = (\lim f)(\lim g)$

Quotient:  $\lim(f/g) = \lim f \div \lim g, \lim g \neq 0$

Composition needs  $g$  continuous at  $\lim f$

Squeeze:  $g \leq f \leq h$  with equal outer limits

## $f$ is continuous at $a$ when

$f(a)$  is defined

$\lim f(x)$  as  $x \rightarrow a$  exists

That limit equals  $f(a)$

Both one-sided limits agree

Failures: removable, jump, infinite

A limit can exist where the value does not.  $\sin(x)/x \rightarrow 1$  as  $x \rightarrow 0$  although  $f(0)$  is undefined.