# Lagrange's Theorem

Mathematics

The divisibility constraint that rules out most candidate subgroups.

**In a finite group, the order of every subgroup divides the order of the group itself.**

$$|G| = |H| \cdot [G : H]$$

**$|G| = 12$** — **Orders must divide 12**

1, 2, 3, 4, 6, 12.

**$|G| = 7$, a prime** — **Only trivial subgroups**

So G is cyclic.

**Order of an element** — **Divides $|G|$**

It generates one.

**THE COMMON MISTAKE**

× **Assuming a subgroup exists for each divisor**

✓ **The converse of Lagrange is false**

$A_4$ has order 12 and no subgroup of order 6. Lagrange restricts which orders can occur; it never produces a subgroup for you.

**Consequences: a raised to $|G|$ is the identity for every a, and every group of prime order is cyclic. Sylow gives a partial converse.**