

The Integrating Factor

Mathematics

The standard method for a first-order linear equation, in order.

For the linear equation $dy/dx + P(x)y = Q(x)$.

1

Put it in standard form

The coefficient of dy/dx must be 1. Divide through if it is not.

2

Compute the factor

$\mu(x) = e$ raised to the integral of $P \, dx$. Drop the constant here.

3

Multiply the equation by μ

The left side then becomes the exact derivative of the product μy .

4

Integrate both sides

μy equals the integral of $\mu Q \, dx$, plus C . The constant belongs here.

5

Divide by μ and check

Solve for y , then substitute back into the original equation.

Worked: $dy/dx + 2y = e^{-x}$ has $\mu = e^{(2x)}$, which gives the solution $y = e^{-x} + C e^{(-2x)}$.