

Inclusion-Exclusion

Mathematics

Counting a union without double counting the overlaps.

Add the sizes, subtract every pairwise overlap, add back every triple overlap, and keep alternating.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A|=10, |B|=8, |A \cap B|=3$$

$$10 + 8 - 3.$$

$$|A \cup B| = 15$$

Three sets

The signs alternate.

$$\sum |A| - \sum |A \cap B| + |A \cap B \cap C|$$

Multiples of 2 or 3 to 30

Overlap is by 6.

$$15 + 10 - 5 = 20$$

THE COMMON MISTAKE

✗ Adding $|A| + |B|$ for overlapping sets

✓ Subtracting the intersection once

Elements lying in both sets are counted twice by the plain sum. Subtracting the intersection restores each of them to exactly one count.

Derangements fall straight out of this: the number of permutations fixing nothing is $n!$ times the alternating sum of $1/k!$.