

Implicit Differentiation

Mathematics

Finding dy/dx for a curve where y is never isolated.

For a curve given by an equation in x and y together.

1 Differentiate both sides

Apply d/dx to every term, treating y as an unknown function of x .

2 Chain rule on every y term

Each y term picks up a factor dy/dx , so $d/dx(y^3) = 3y^2 dy/dx$.

3 Product rule on mixed terms

$d/dx(xy) = y + x dy/dx$. Any term in both variables needs it.

4 Collect the dy/dx terms

Move every term containing dy/dx to one side, the rest to the other.

5 Factor and divide

Factor out dy/dx and divide. The answer usually involves x and y both.

Worked: $x^2 + y^2 = 25$ gives $2x + 2y dy/dx = 0$, so $dy/dx = -x/y$ — undefined at $y = 0$, where the tangent is vertical.