Ideals & Quotient Rings

Mathematics

What an ideal absorbs, and which ones give domains, which fields.

Constructing R/I for a ring R and an ideal I.

1. **Check I really is an ideal**
   It is an additive subgroup, and rI stays inside I for every r in R.

2. **Form the cosets**
   Elements of R/I are r + I. Two agree when their difference lies in I.

3. **Define both operations**
   Add and multiply representatives, then take the coset of the result.

4. **Classify the ideal**
   R/I is an integral domain iff I is prime, and a field iff I is maximal.

5. **Use the construction**
   ▯[x] modulo (x^2 + 1) is isomorphic to ▯. New rings from old.

An ideal absorbs multiplication from both sides. A subring need not, which is exactly why ideals and not subrings give quotients.