Group Homomorphisms

Mathematics

Structure-preserving maps, and the kernel test for injectivity.

A homomorphism preserves the operation: the image of a product is the product of the images.

$$\varphi(ab) = \varphi(a)\varphi(b) \text{ for all } a, b \in G$$

φ of the identity — **The identity of H**

Follows from the axiom.

The kernel of φ — **A normal subgroup of G**

Normal, always.

Kernel is trivial — **φ is injective**

The standard test.

THE COMMON MISTAKE

× Checking injectivity element by element

✓ Checking the kernel is trivial

$\varphi(a) = \varphi(b)$ is equivalent to ab^{-1} lying in the kernel, so one kernel computation settles injectivity for the entire group.

The image is a subgroup of the codomain and the kernel is a normal subgroup of the domain. Both facts come free.