

Group Axioms

Mathematics

The four axioms, and the facts that follow before you prove anything else.

A set G with one associative operation, an identity and inverses.

The four axioms

Closure: ab lies in G for all a, b in G

Associativity: $(ab)c = a(bc)$

Identity e with $ea = ae = a$

Inverses: each a has a^{-1} with $aa^{-1} = e$

What follows at once

The identity is unique

Each inverse is unique

$(ab)^{-1} = b^{-1}a^{-1}$

Cancellation: $ab = ac \Rightarrow b = c$

Abelian adds $ab = ba$, and is extra

Commutativity is not an axiom. Matrix multiplication and function composition are the standard non-abelian examples.