

Graph Vocabulary

Mathematics

The definitions and counting facts every graph question assumes.

A graph is a set of vertices together with a set of edges, each joining a pair of them.

EDGES IN A COMPLETE GRAPH

$$n(n - 1)/2$$

EDGES IN A TREE

$$|V| - 1$$

SUM OF ALL DEGREES

$$2|E|$$

- Degree of a vertex is the number of edges at it
- Handshake lemma: the degrees sum to twice $|E|$
- A tree is connected and acyclic, so $|E| = |V| - 1$
- Bipartite exactly when there is no odd cycle
- The complete graph on n vertices has $n(n - 1)/2$ edges
- A simple planar graph obeys $|E| \leq 3|V| - 6$
- Euler: $|V| - |E| + |F| = 2$ when connected and planar

LOOK FOR IT

The number of odd-degree vertices in any graph is always even.

Kuratowski: a graph is planar exactly when it contains no subdivision of K_5 or of the complete bipartite $K(3, 3)$.