

# Gram-Schmidt

## Mathematics

Turning any basis into an orthonormal one spanning the same space.

*Subtract off what is already accounted for, then normalise.*

1

### Keep the first vector

Set  $w_1 = v_1$ . The first direction is carried over unchanged.

2

### Subtract the projection

$w_2 = v_2$  minus the projection of  $v_2$  onto  $w_1$ . Now  $w_2 \perp w_1$ .

3

### Repeat for each vector

Subtract the projection onto every  $w$  constructed so far.

4

### Normalise

Divide each  $w$  by its own length to get unit vectors.

5

### Check the result

Every pair has dot product 0 and every vector has length 1.

The projection of  $v$  onto  $w$  is  $(v \cdot w / w \cdot w)w$ . The classical algorithm loses orthogonality numerically; the modified version is used instead.