

Completing the Square

10th Grade

Turning any quadratic into vertex form — where the formula comes from.

Add and subtract $(b/2)^2$ to turn $x^2 + bx$ into a perfect square, leaving the equation unchanged in value.

$$x^2 + bx + (b/2)^2 = (x + b/2)^2$$

$$x^2 + 6x$$

$$(x+3)^2 - 9$$

Half of 6 is 3; 3^2 is 9.

$$x^2 + 6x + 5 = 0$$

$$(x+3)^2 = 4$$

$x = -1$ or -5 .

$$x^2 - 8x + 3$$

$$(x-4)^2 - 13$$

Half of -8 is -4 .

$$2x^2 + 8x + 1$$

factor out the 2 first

$2(x^2 + 4x) + 1$, then complete inside.

THE COMMON MISTAKE

✗ adding $(b/2)^2$ without subtracting it

✓ **add AND subtract it**

Adding alone changes the expression's value. It must net to zero.

Applying this to $ax^2 + bx + c = 0$ in general produces the quadratic formula. That is where it comes from.