

Gaussian Elimination

Mathematics

Row reduction, and how to read the answer off the echelon form.

Three row operations, applied until the shape is triangular.

1

Form the augmented matrix

Write $[A \mid b]$. Every row operation acts across the whole row.

2

Get a leading 1

Swap rows if you must, then scale so the pivot entry is 1.

3

Clear below the pivot

Subtract multiples of the pivot row from every row beneath it.

4

Move down and right

Repeat on the submatrix until the matrix is in row echelon form.

5

Clear above, or back substitute

Clearing above each pivot too gives reduced row echelon form.

6

Read the solution

Pivot columns give basic variables; the rest are free parameters.

A row reading $0 = 1$ means no solution. Fewer pivots than unknowns means infinitely many. Only three outcomes are possible.