Fundamental Theorem

Mathematics

The two halves that make differentiation and integration inverse operations.

If f is continuous, then G(x) = the integral of f from a to x is differentiable with G′(x) = f(x).

∫ from a to b of f(x) dx = F(b) − F(a)

d/dx of ∫ 0 to x of t^2 dt — **x^2**

Part one, directly.

∫ from 0 to 1 of 2x dx — **1**

F(1) − F(0), F = x^2.

Upper limit is x^2 — **$f(x^2) \cdot 2x$**

Chain rule as well.

THE COMMON MISTAKE

× Using F(b) − F(a) across a discontinuity

✓ Splitting the interval at the bad point

The integral of $1/x^2$ from −1 to 1 is not −2. The integrand blows up at 0, and the theorem needs an antiderivative on the whole interval.

Part two needs f to be integrable and F to be an antiderivative on all of [a, b]. Continuity of f delivers both.