

Fourier Series

Mathematics

The coefficient formulas, the parity shortcuts, the convergence limits.

A periodic function written as a sum of sines and cosines.

The coefficients

$$f(x) = a_0/2 + \sum (a_n \cos nx + b_n \sin nx)$$

a_n from integrating $f(x) \cos nx$ over a period

b_n from integrating $f(x) \sin nx$ over a period

$a_0/2$ is simply the mean value of f

Period $2L$: replace every nx by $n\pi x/L$

Shortcuts and limits

If f is even, only cosine terms survive

If f is odd, only sine terms survive

At a jump the series gives the midpoint

Gibbs overshoot near a jump is about 9%

Parseval relates $\sum a_n^2$ to the integral of f^2

Convergence is pointwise for a piecewise smooth f , but never uniform across a jump. The Gibbs overshoot does not shrink with more terms.