

Fourier Series

Electrical Engineering

Periodic signals as sums of harmonics, and what smoothness does to them.

Any periodic signal is a sum of harmonics of its own fundamental frequency.

$$x(t) = \sum c_k \exp(j 2\pi k f_0 t) \cdot f_0 = 1/T$$

Square wave

Slow decay.

Odd harmonics, as $1/k$

Triangle wave

Much faster decay.

Odd harmonics, as $1/k^2$

Half-wave symmetry

Symmetry test.

Even harmonics vanish

THE COMMON MISTAKE

✗ **Expecting the series to converge at a jump**

✓ **Accepting about 9% overshoot there**

Partial sums overshoot a discontinuity by a fixed fraction no matter how many terms are added. That is the Gibbs phenomenon.

Smoothness sets decay: the more derivatives a waveform has, the faster its harmonic amplitudes fall away.